Navarra Center for International Development

Working Paper nº 07/2015

FRACTIONAL INTEGRATION AND ASYMMETRIC VOLATILITY IN EUROPEAN, ASIAN AND AMERICAN BULL AND BEAR MARKETS. APPLICATIONS TO HIGH FREQUENCY STOCK DATA

Luis A. Gil-Alana
NCID, University of Navarra, Pamplona, Spain

OlaOluwa S. Yaya
University of Ibadan, Ibadan, Nigeria

Olarenwaju I. Shittu
University of Ibadan, Ibadan, Nigeria

Navarra Center for International Development
WP-07/2015

Fractional Integration and Asymmetric Volatility in European, American and Asian Bull and Bear Markets: Application to High Frequency Stock Data

Luis A. Gil-Alana, OlaOluwa S. Yaya and Olarenwaju I. Shittu

Working Paper No. 07/2015
April 2015

ABSTRACT

This paper is a follow up to Gil-Alana, Shittu and Yaya (2014). In that paper, fractional integration and symmetric volatility modeling were considered on monthly frequency data, while the present paper considers high frequency data on an asymmetric volatility model. The data were first identified within the respective bull and bear phases following earlier results in the previous paper. Then, fractional integration and the asymmetric volatility model of Glosten, Jaganathan and Runkle (GJR) were applied on the stock returns. Long range dependence was detected in the squared stock returns at each market phase, and they were more persistent than those obtained in the monthly frequency data. The estimates of asymmetry of the GJR model actually detected the different patterns of the bad news (bear phases) and the good news (bull phases).

Keywords: Bull and bear periods; fractional integration; high frequency; stock returns; volatility

JEL Classification: C22; G14; G15

Corresponding author: Luis A. Gil-Alana
Department of Economics & ICS_NCID
Universidad of Navarra
31080 Pamplona
Spain

1. Introduction

This paper deals with modeling asymmetry and volatility in the bull and bear stock markets in the US, Europe and Asia using high frequency market data. In fact, the paper is a follow up to Gil-Alana *et* al. (2014), which identified stock indices in the US, Europe and Asia in four different market phases. The stock markets examined were the US stock markets (Dow Jones Industrial, Nasdaq and Standard & Poor 500), European markets (CAC, DAX and FTSE) and Asian (Nikkei, Hang Seng and STI) indices. In the paper, monthly data were used and the identified turning points were used as key to identify the corresponding bull and bear phases. In the present work, we apply high frequency (daily) data instead. We check for the degree of persistence in the returns during each bull and bear phase using the fractional differencing parameter as a measure of persistence in the squared returns, and using non-parametric, semiparametric and parametric methods of long range dependence. We then proceed to study the volatility and leverage effect in each market phase using the Glosten, Jaganathan and Runkle–Generalized Autoregressive Conditional Heteroscedastic (GJR-GARCH) model of Glosten *et* al. (1993), which captures the asymmetry in the volatility of the asset returns.

Volatility clustering is a phenomenon where large changes in returns are followed by large changes of either sign in the same return series, and small changes tend to be followed by small changes of the returns. The asset returns are highly peaked (leptokurtic) and slightly asymmetric. Asymmetric and leverage effects were first noted by Black (1976) in stock prices data. Negative returns (price decreases) tend to increase volatility to a larger extent than positive returns (price increases) of the same magnitude (Francq and Zakoian, 2010) and the characterization of the volatilities in stocks as bull and bear phases is important for describing the behavior of stock markets.

The bull and bear phases are general terms used to describe increases and decreases in stock markets (Ramos et al, 2011). Maheu and McCurdy (2000) stated that as the bull market persists, investors are more optimistic about the future, and invest more in stocks, which implies that the probability of switching out from the bull market decreases with time. A general rule is that a stock is in the bull phase if it has increased more than 20% over a period of time and it is in the bear phase if it has fallen around 20% (Gomez-Biscarri and Perez de Gracia, 2004). There are few papers on volatility in the bull and bear phases and the majority of them found volatility to be higher during bear markets than in bull periods (Maheu and McCurdy, 2000; Gomez-Biscarri and Perez de Gracia, 2004; Jones et al., 2004; González et al., 2005; Guidolin and Timmermann, 2005; Tu, 2006; Casarin and Trecroci, 2006; Cunado et al., 2008). Some of these papers studied stock market volatility using long range dependence techniques. Gil-Alana et al. (2014) obtained similar results to the previous authors and showed that persistence is a feature of the volatility of the returns, and the half-life estimates on the GARCH model could not shed further light on the remaining asymmetry in each of the identified market phases.

Still within this context of volatility persistence, some authors have applied GARCH models to study the volatility in each of the market phases in just one economy. However, for the sake of comparison it is also important to study the global markets together. Asymmetry is an important issue in the context of stock markets and the GARCH-type of models may provide a good representation of volatility. Pagan and Sossounov (2003) considered monthly prices of the S&P500 index between 1837 and 1997 and applied a battery of models including the random walk, the GARCH, the Exponential GARCH (EGARCH) and a hidden layer Markov chain model on the data, and found that the EGARCH model provides the best match to most of the market phase characteristics. Others such as Awartani and Corradi (2005) examined the relative predictive ability of different GARCH models with particular emphasis on the predictive content of the asymmetric components. They found that for one-step ahead pair-wise comparisons, the GARCH(1,1) model is beaten by the asymmetric

GARCH models in the analysis of daily closing prices of the S&P500 index from 1990 to 2001. The same finding was observed for multiple comparisons, though the predictive superiority of the asymmetric model was not so strong as in the one-step ahead case. Huang (2011) found the GJR-GARCH model to outperform other volatility models including GARCH when applied on S&P 500 stock index returns. Fractional integration models have also been employed in the context of GARCH models by means of the Fractionally Integrated GARCH (FIGARCH) model proposed by Baillie, Bollerslev and Mikkelsen (1996). Chung (1999) proposed a modified version of the model known as FIGARCHC to correct the fractional differencing problem on the constant term. Davidson (2004) proposed the generalized version of the FIGARCH model which nests the FIGARCH model. The idea of FIGARCH modelling is also extended to modelling both long memory and structural changes in the conditional variance in the Adaptive-FIGARCH (A-FIGARCH) model of Baillie and Morana (2009). The FIGARCH model is applied in Beine, Bénassy-Quéré and Lecourt, (2002) and Vilasuso (2002) on exchange rate volatility modelling. This has also been considered in Xekalaki and Degiannakis (2010) in studying the volatility of international stocks, but due to the fact that the model is symmetric, it never emerged as the best among the other asymmetric models since the dynamic of stock time series is asymmetric.

This paper further establishes the asymmetric volatility in the daily stock prices and attempts to classify the prices of these stocks into market phases. The outline of the paper is as follows: Section 2 briefly describes the methodology employed in the paper. Section 3 presents the data and the main empirical results, while Section 4 contains some concluding comments.

2. Methodology

2.1 Fractional integration and estimation approaches

We define a fractionally integrated model as

$$(1 - B)^d x_t = u_t , \qquad t = 1, 2, ...$$
$$x_t = 0, \qquad t \leq 0, \qquad (1)$$

where B is the backward shift operator; x_t is the observed time series, supposed to be fractionally differenced, *d* is the fractional differencing parameter, and u_t is the resulting covariance stationary I(0) process. An I(0) process is defined as a covariance stationary process with a spectral density function that is positive and finite at any frequency in the spectrum. This is also denominated "*short memory*" as opposed to the case of "long memory" that takes place when d > 0 in the I(d) contexts. Thus, fractional integration belongs to the category of "l*ong memory*" processes. On the other hand, in the applied work, long memory has been frequently associated to the volatility of financial assets.

The estimation of the fractionally differencing parameter is carried out first using the non-parametric approach of Lo (1991) which applied the Hurst (1951) rescaled R/S statistic; then we also use a Whittle parametric method (Dahlhaus, 1989) along with a parametric testing procedure suggested by Robinson (1994) and finally, a semiparametric (local) Whittle estimate (Robinson, 1995). All these methods except Robinson (1994) are sensitive to estimating the differencing parameter within the stationary range, i.e. $-0.5 \leq d < 0.5$, whereas the parametric approach of Robinson (1994) allows testing any real value of d including then, stationary ($d < 0.5$) and also nonstationary ($d \geq 0.5$) hypotheses.[1]

The non-parametric approach of Lo (1991) is based on a rescaled range statistic (R/S) defined as:

$$R/S = \frac{1}{S_N}\left(\sup_{1\leq m\leq N} \sum_{j=1}^{m}(y_j - \bar{y}) - \inf_{1\leq m\leq N} \sum_{j=1}^{m}(y_j - \bar{y}) \right), \qquad (2)$$

where S_N is the standard deviation from $S_N^2 = \frac{1}{N}\sum_{j=1}^{N}(y_j - \bar{y})$. The value of S_N is positive

[1]Teverovsky et al. (1999) suggest using the non-parametric approach of Lo (1991) along with other testing procedures.

because the deviations from the sample mean sum up to zero, then the supremum and infimum of the partial sums are positive and negative respectively. The probability limit was shown to be constant, that is $\Pr ob\left[\lim_{N\to\infty}\left(\frac{R/S}{T^H S_t}\right)\right]=c$, where c is the critical value, and H is the Hurst coefficient, estimated by,

$$\hat{H} = \frac{1}{\log(N)}\log(R/S). \qquad (3)$$

From (3), $\log(N)=2\log(R/S)$ is an indication for short memory (stationary I(0) series) and $\log(N)>2\log(R/S)$ is an indication for long memory (I(d), d > 0). The fractional differencing parameter, d is then given as,

$$\hat{d} = \hat{H} - 0.5, \qquad (4)$$

which is a value computed in the long memory range. Lo (1991) then found out that the R/S statistic might be very sensitive to short memory series and proposed a modified rescaled range statistic which adjusts the autocorrelation structure. The R/S statistic in (2) is then re-defined with $S_N = S_N(q)$, where $S_N(q) = \left(S_N^2 + 2\sum_{J=1}^{Q} w_j(q)\hat{\gamma}_j\right)^{1/2}$, and $w_j = 1 - j/(q+1)$ such that $q < N$. The ML standard deviation is estimated by S_N and the j^{th} -order sample autocovariance by $\hat{\gamma}_j$.

In the context of semiparametric methods, we propose an estimate of d developed by Robinson (1995). This method is essentially a local 'Whittle estimator' in the frequency domain, which uses a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d\frac{1}{m}\sum_{s=1}^{m}\log\lambda_s\right), \qquad (5)$$

$$\overline{C(d)} = \frac{1}{m} \sum_{s=1}^{m} I(\lambda_s) \lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{N}, \qquad \frac{m}{N} \to 0,$$

where m is a bandwidth number, and $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) = \frac{1}{2\pi N} \left| \sum_{t=1}^{T} x_t \, e^{i\lambda_s t} \right|^2,$$

and $d \in (-0.5, 0.5)$. Under finiteness of the fourth moment and other mild conditions, Robinson (1995) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad \text{as } N \to \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroscedasticity, which is supposed to be very useful for the data used in our application, and is more efficient than other more recent semiparametric competitors (Robinson and Henry, 1999).[2]

Finally, we also employ parametric approaches, including a Whittle method in the frequency domain and an LM test described in Robinson (1994). This latter model considers the following model,

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots \tag{6}$$

with

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{7}$$

where y_t refers once more to the observed data, and it tests the fractional differencing parameter d (i.e., H_o: $d = d_o$ for any real d_o) for the three standard cases appearing in the literature, i.e., the cases of no regressors (i.e., $\beta_0 = \beta_1 = 0$ a priori in (6)), an intercept (β_0 unknown and $\beta_1 = 0$ a priori), and an intercept with a linear time trend (i.e., β_0 and β_1

[2] Further refinements of the estimation approach are given in Robinson and Henry (1999), Velasco and Robinson (2000), Phillips and Shimotsu (2004), Shimotsu and Phillips (2005) and Abadir et al. (2007).

unknown). Since this method is parametric we need to specify a functional form for the I(0) error term u_t. Here, we will present the results based on both uncorrelated (white noise) and correlated errors. In the latter case we use the exponential spectral model of Bloomfield (1973), which is a non-parametric approach of modeling the I(0) error term that produces autocorrelations decaying exponentially as in the AR(MA) case. (See, Gil-Alana, 2004, for an overview of this approach in the context of fractional integration). Another advantage of this approach is that Gaussianity is not required with a moment condition only of order 2 be required.

2.2 The asymmetric volatility model

Asymmetric volatility refers to the inverse correlation between stock index returns and volatility (Bekaert and Wu, 2000; Wu, 2001). A drop in the value of a stock (negative return) increases the financial leverage, this makes the index riskier and thus increases its volatility. (Black, 1976). Since stock indices are usually measured in terms of returns, we consider the stock market returns series, y_t as:

$$y_t = \log P_t - \log P_{t-1} = \log\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t refers to the price indices. We examine the absolute return, $\left(|y_t|\right)$ and squared return $\left(y_t^2\right)$ series (denoted below as V_t), used as proxies for the volatility[3], under the assumption that they follow a long memory process of the form

$$(1-L)^d V_t = c_0 + \sum_{i=1}^{p} c_i (1-L)^d V_{t-i} + \varepsilon_t , \qquad (8)$$

[3]Absolute returns has been used as proxies for the volatility among others by Ding et al. (1993), Granger and Ding (1996), Bollerslev and Wright (2000), Gil-Alana (2005), Cavalcante and Assaf (2004), Sibbertsen (2004) and Cotter (2005), whereas squared returns were used in Lobato and Savin (1998), Gil-Alana (2003), Cavalcante and Assaf (2004) and Cotter (2005).

where the parameters c_0 and c_i are the constant and AR parameters in the model, and the estimated differencing parameter d is obtained using the methods presented above.

The residuals ε_t in (8) may be described as $\varepsilon_t = \sigma_t z_t$ where the conditional volatility process σ_t follows the model specification of Ding et al.(1993) which uses an indicator function to predict the positive and negative returns. The GARCH innovations z_t follows a skewed Student t-distribution described in (11) below. Ding et al (1993) state that the volatility of an asset price with returns given in (7) is,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \left[\gamma_i \, d(\varepsilon_{t-j} < 0) \ \varepsilon_{t-i}^2 \right] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 , \quad (9)$$

which is known as the Glosten, Jaganathan and Runkle (GJR) model of order (*p*,*q*). The α_i (i = 1, ..., p) and the β_j (j = 1, ..., q) are the ARCH and GARCH parameters for the ARCH term ε_{t-1}^2 and GARCH term σ_{t-1}^2 respectively, where γ_i (i = 1, ..., p) are the additional parameters to be estimated. The model allows good news ($\varepsilon_{t-i} > 0$) and bad news ($\varepsilon_{t-i} < 0$) to have different effects on the conditional variance. The indicator function d(x) is defined such that $d(\varepsilon_{t-i} < 0) = 1$ if $\varepsilon_{t-i} < 0$ and $d(\varepsilon_{t-i} > 0) = 0$ if $\varepsilon_{t-i} > 0$. For example, in the case of GJR(1,1) model, good news has an impact of $\alpha_1 + \beta_1$, while bad news has an impact of $\alpha_1 + \beta_1 + \gamma_1$.

2.3 Estimation and distributional assumption

The estimation of the GARCH model may be based on normal, t-Student and Generalized Error Distribution (GED) distributional assumptions. Recently, skewed versions of these distributions have been proposed. This paper applies the skewed t-Student distribution proposed in Lambert and Laurent (2000, 2001), which accounts for both fat left and right tails of the innovations ε_t. It is given by

$$f(\varepsilon_t, v, g) = \frac{\Gamma\left[(v+1)/2\right]}{\sigma_t \Gamma(v/2)\sqrt{\pi(v-2)}} \left(\frac{2s}{g+g^{-1}}\right) \left[1 + \frac{s\varepsilon_t + m}{\sigma_t^2 (v-2)} g^{-\Pi_t}\right]^{-(v+1)/2} \quad (11)$$

where the degree of freedom in distribution $\nu > 2$, g is the asymmetry parameter, $\pi_t = 1$ if $\varepsilon_t \geq -ms^{-1}$, and $\pi_t = -1$ if $\varepsilon_t < -ms^{-1}$, $m = \Gamma\left[(v-1)/2\right]\sqrt{(v/2)}\left[\Gamma(v-2)\sqrt{\pi}\right]^{-1}(g-g^{-1})$ and $s = \sqrt{g^2 + g^{-2} - m^2 - 1}$. The Quasi Maximum Log-Likelihood (QML) of this distribution is then simplified using the MaxSA algorithm of Goffe, et al. (1994) implemented in GARCH program by Laurent (2007) and Laurent and Peters (2006).

3. Data and empirical results

The datasets used in this work are daily US, European and Asian open stock market indices. They are FTSE, CAC40 and DAX for the European market, Standard and Poor, Nasdaq and Dow Jones for the US; and Nikkei, Hang Seng and STI for the Asian markets. The data span from 13 March 2002 to 16 May 2012. These were retrieved from the Yahoo Finance website: http://finance.yahoo.com.

The identified month for the market phase dates used in Gil-Alana et al. (2014) are used to identify the peak and trough periods. In that paper, monthly data were used while in this paper, we are using daily data in order to increase the data sample points and to avoid loss of information. The exact bull and bear periods for the daily frequency are identified following Pagan and Sossounov (2003) and Gil-Alana et al. (2014). Table 1 displays the results of the series identification.

[Insert Table 1 about here]

We observe, in Table 1, that for each of the stocks in Europe, America and Asia, the market troughs and peaks are different. Actually, we assume the same starting and ending point for the data and, for example, in the European FTSE stocks, the first trough is 1st October 2002 and the corresponding points for CAC40 and DAX are 22nd October 2002 and 21st October 2002. In these three markets, the market peak points are 15th November 2007 for both FTSE and DAX and 23rd November 2007 for CAC40. The second market troughs for

FTSE, CAC40 and DAX are 20th March 2009, 16th March, 2009 and 30th March, 2009 respectively. We see that the three European markets behave in a similar way. The stock market phases in America indicate that the first trough is the same for the S&P and the Nasdaq stocks (1st November 2007) and the peak period (9rd March 2009) is the same for Nasdaq and Dow Jones stocks. In the Asian market, none of the market troughs and peaks are the same for the three stocks (Nikkei, Hang Seng and STI). This may suggest that stocks in Asia do not respond sharply to good or bad news.

Table 2 presents the results of the descriptive measures of stocks by bull and bear market phases. We see that the average returns at the bear phases (1st and 3rd market phases) are negative and the averages at the bull phases (2nd and 4th market phases) are positive. The bull phase is the period of good news when stocks increase and returns are mostly positive. The bear phase is the period of bad news when stocks decrease and returns are mostly negative. In terms of magnitude, the stocks in Europe and America present market returns that are greater in the 1st bear phase than in the next bear phase, whereas in the Asian market, Hang Seng and STI have their market returns in the 2nd bear phase greater than in the previous bear phase. Relating the mean returns with the median estimates, we expect significant non-normality in the market returns. The estimates of skewness for each phase actually suggest asymmetric volatility models for the stock returns.

[Insert Table 2 about here]

Next we focus on the measure of persistence by looking at the degree of integration of each return phase. Table 3 displays the Whittle estimates of d using a parametric approach where the error term is supposed to be white noise. We also display in the table the confidence band of the non-rejection values of d using Robinson's (1994) approach. We present the estimates and the bands for the three standard cases of no regressors, an intercept, and an intercept with a linear time trend.

[Insert Table 3 about here]

As expected most of the estimates are very close to 0 suggesting that returns are I(0); however we also observe some cases with estimates which are strictly below 0 implying that the underlying (log-)prices are mean reverting. This happens for the 3rd and 4th phases for the FTSE and the CAC40 in the European markets; for the US case, mean reverting prices are obtained in the 2nd and 3rd phases for the S&P; 1st and 3rd (bear periods) for the Nasdaq, and 2nd, 3rd and 4th phases in the Dow Jones. Finally, for the Asian case, mean reversion is obtained in the 1st, 2nd and 3rd phase for the Hang-Seng, and in the 2nd phase (bull) for the STI. Nevertheless these negative estimates of d are very small and close to zero suggesting that a very small degree of mean reversion occurs in these cases with shocks disappearing very slowly.

[Insert Table 4 about here]

Next we look at the results based on the semiparametric method of Robinson (1995). They are presented in Table 4 for the case of $m = (N)^{0.5}$, which is the bandwidth usually employed in the empirical works. Here, we obtain evidence of mean reverting prices (estimates of d in the returns significantly below 0) in the 1st and 3rd phases of CAC40, 3rd phase in the DAX; 1st phase in the Nasdaq, and 2nd and 4th phases in the Dow Jones. For the Asian markets, evidence of mean reversion is obtained in the 4th phase of the Nikkei, 3rd phase in the Hang Seng, and 1st phase in the STI.

The results based on the non-parametric approach (Lo, 1991), in Table 5, are quite similar to those obtained with the parametric and semiparametric approaches in Tables 3 and 4.

[Insert Table 5 about here]

Table 6 displays the estimates of d in the squared returns and most of them are significantly positive, corroborating the fact that they display long memory behavior. In fact, the only cases where the I(0) (short memory or d = 0) hypothesis cannot be rejected correspond to the Hang Seng and STI indices. Table 7 displays the estimates based on the semiparametric approach and except for the first phase of the STI index displays long memory behavior (d > 0) throughout. Table 8 displays the estimates based on the non-parametric approach and we found very similar estimates to those presented above for the parametric case in Table 6. The estimates of the differencing parameter based on the squared returns and computed with the semiparametric approach in Table 7 seem to be slightly over-estimated.

[Insert Tables 6, 7 and 8 about here]

Tables 9, 10 and 11 present the results of the estimated GJR models for the three stock markets. Using the MaxSA algorithm[2] of Goffe et al. (1994) in the OxGARCH program, we found the asymmetric GJR model as the best plausible model with which to estimate the stock returns across the phases of the stock markets. First order autocorrelation is found to be significant in some of the stock returns and we therefore estimated AR-GJR for all the market phases in order to allow uniformity.

[Insert Tables 9, 10 and 11 about here]

In the European market (Table 9) for example, the estimates of GARCH parameters $(\hat{\beta}_1)$ are quite similar to those obtained in Gil-Alana et al. (2014). The condition of positivity of the ARCH parameter $(\hat{\alpha}_1)$ was relaxed in the GJR model and some of the estimates were computed as negative, which allowed the existence of the asymmetric parameter, (γ_1). In this market, the asymmetric parameter estimates are significant at 1st, 2nd and 3rd market phases across the three stocks (FTSE, CAC40 and DAX). At the 4th bull phase, though the estimates

[2]The MaxSA algorithm in OxGARCH software was run based on the assumption of skewed Student t distribution of GARCH innovations.

are quite high, but only being significant in the case of the CAC40. Similar results are obtained for the American and the Asian stock markets (Tables 10 and 11).

Regarding the asymmetry in the returns at the identified market phases, it should remembered that the bear phases (1st and 3rd) present bad news periods (when returns are negative) and good news periods (when returns are positive). In the European market (Table 9), at 1st phase (bear), the average returns are negative (bad news, see Table 2) and the estimates of bad news impact ($\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$) are greater than those of good news impact ($\hat{\alpha}_1 + \hat{\beta}_1$) on the conditional volatility. This tells us the extent to which bear markets (bad news) affect the reactions of the entire market. At the 2nd phase (bull), which is a good news period, as expected, the estimates of the impact of good news increased while those of bad news dropped significantly. At the 3rd phase (bear), the bad news impacts on the conditional volatility are higher than those obtained at the 1st phase (bear). For this stock, with bad news, the estimates of the impact of bad news are higher than those obtained in the 2nd phase (bull), where there is good news. At the 4th phase (bull), there is good news and the impact of good news increased while the impact of bad news fell compared with that obtained in the previous market phase.

Similar results on the impacts of bad news and good news are obtained for the American and the Asian markets in Tables 10 and 11 with the exception of the Hang Seng and the STI in the Asian market where the GJR model did not converge and resulted in spurious estimates which were not reported here.

4. Concluding remarks

In this paper, we have examined the degree of persistence in the returns and the squared returns as well as the asymmetry in volatility of high frequency market stock data from the European, the US and the Asian markets. We examined the FTSE, the CAC40 and the DAX in Europe; in the US, the S&P, the Nasdaq and the Dow Jones and in Asia, the Nikkei, the

Hang Seng and the STI. The exact turning points to judge the market phases were identified based on previous results obtained in Gil-Alana et al. (2014). We have applied fractionally integrated techniques in the larger daily dataset, and obtained substantially more reliable results in comparison to Gil-Alana et al. (2014), who applied monthly frequency data. The results based on daily data are more persistent across the identified market phases in the sense that the orders of integration are slightly higher than those obtained in Gil-Alana et al.'s (2004) and based on monthly data. The analysis conducted in this work is relevant in the sense that fractional integration has been criticized by some authors saying that it might be an artificial artifact generated by the presence of structural breaks, which have not been taken into account (Diebold and Inoue, 2001; Granger and Hyung, 2004; Starica and Granger, 2005; Davidson and Sibbertsen, etc.). In this context the analysis of fractional integration and the asymmetric volatility in the bull and bear markets may at least partially solve this controversy. Other approaches of fractional integration dealing with structural breaks can also be considered (Gil-Alana, 2008; Baillie and Morana, 2009; Ohanissian et al.; Perron and Qu, 2010; Qu, 2011, and Shao, 2011; etc.).

The estimates from the GJR models actually confirmed the asymmetry in the market phases (bull and bear), with the bear periods having more (significant) impacts on the conditional volatility of the stocks returns. Other papers (see Awartani and Corradi, 2005; Pagan and Sossounov, 2003; Huang, 2011, etc.) have established asymmetry in US, UK and Asian stock indices. We can then extend this work by looking at the forecasting ability of the volatility series at each of the identified bull and bear phases. Secondly, we can consider the Smooth Transition-GARCH (ST-GARCH) model of Hagerud (1997) and González-Rivera (1998) which captures the regime switching dynamics in the stocks volatility and compared the results with those obtained for the case of GJR-GARCH model.

References

Abadir, K.M., Distaso, W. and Giraitis, L. (2007). Nonstationarity-extended Local Whittle Estimation. Journal of Econometrics, 141, 1353-1384.

Awartani, B.M.A. and Corradi, V. (2005). Predicting the volatility of the S&P-500 stock index via GARCH models: the role of asymmetries. International Journal of Forecasting 21, 167-183.

Baillie, R. T., T. Bollerslev and H.O. Mikkelsen (1996), Fractionally integrated generalized autoregressive conditional heteroscedasticity, Journal of Econometrics 74, 3-30.

Baillie, R.T. and C. Morana (2009) Modelling long memory and structural breaks in conditional variances. An adaptive FIGARCH approach, Journal of Economic Dynamics and Control 33, 1577-1592.

Beine, M., Bénassy-Quéré, A. and Lecourt, C. (2002). Central bank intervention and foreign exchange rates: New evidence from FIGARCH estimations. Journal of International Money and Finance, 21, 115-144.

Bekaert, G., and Wu, G. (2000). Asymmetric volatility and risk in equity markets. Review of Financial Studies 13, 1–42.

Black, F. (1976). Studies of stock market volatility changes. Proceedings of the American Statistical Associations, Business and Economic Statistics Section, 177-181.

Bloomfield, P. (1973). An exponential model in the spectrum of a scalar time series. Biometrika 60, 217-226.

Bollerslev, T., Wright, J.H., 2000, High frecuency data, frequecy domain inference and volatility forecasting, Review of Economics and Statistics 83, 596-602.

Casarin, R. and Trecroci, C. (2006). Business cycle and stock market volatility: A practical filter approach. Cahier du CEREMADE N. 0610, University of Paris Dauphine.

Cavalcante, J., Assaf, A., 2004, Long range dependence in the returns and volatility of the Brazilian stock market, European Review of Economics and Finance 3, 5-22.

Chung, C.F. (1999). Estimating the fractionally integrated GARCH model. National Taiwan University. Working Paper.

Cotter, J., 2005, Uncovering long memory in high frequency UK futures, European Journal of Finance 11, 325-337.

Cunado, J., Gil-Alana, L.A. and Perez De Gracia, F. (2008). Stock market volatility in US Bull and Bear markets. Journal of Money, Investment and Banking 1, 24-32.

Dahlhaus, R (1989). Efficient parameter estimation for self-similar process, Annals of Statistics 17, 1749-1766.

Davidson, J. (2004). Moment and memory properties of linear conditional heteroscedasticity models, and a new model. Journal of Business and Economic Statistics, 22(1), 16-29.

Davidson, J. and Sibbertsen, P., (2005). Generating schemes for long memory processes: Regimes, aggregation and linearity. Journal of Econometrics128, 253.282.

Diebold, F. and A. Inoue, 2001. Long memory and regime switching. Journal of Econometrics 105, 131-159.

Ding, Z., Granger, C.W.J and Engle, R.F. (1993). A long memory property of stock markets and a new model. Journal of Empirical Finance 1, 83-106.

Francq, C. and Zakoian, J-M. (2010). GARCH models: Structure, Statistical inference and financial applications. John Wiley and Sons Ltd.

Gil-Alana, L.A., 2003, Fractional integration in the volatility of asset returns, European Review of Economics and Finance 2, 41-52.

Gil-Alana, L.A. 2004. The use of the Bloomfield (1973) model as an approximation to ARMA processes in the context of fractional integration. Mathematical and Computer Modelling 39, 429-436.

Gil-Alana, L.A., 2005, Long memory in daily absolute and squared returns in the Spanish stock market, Advances in Investment Analysis and Portfolio Management 1, 198-217.

Gil-Alana, L.A., 2008, Fractional integration and structural breaks at unknown periods of time, Journal of Time Series Analysis 29, 163-185.

Gil-Alana, L.A., Shittu, O.I. and Yaya, O.S. (2014). On the Persistence and Volatility in European, American and Asian Stocks Bull and Bear Markets. Journal of International Money and Finance 40, 149-162.

Glosten, L.W. Jaganathan, R. and Runkle, D.E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48, 1779–1801.

Goffe, W.L., Ferrier, G.D. and Rogers, J. (1994). Global optimization of statistical functions with simulated annealing. Journal of Econometrics 60, 65–99.

Gomez Biscarri, J. and Perez de Gracia, F. (2004). Stock market cycles and stock market development in Spain, Spanish Economic Review 6, 127-151.

González, L., Powell, J.G., Shi, J. and Wilson, A. (2005). Two centuries of bull and bear market cycles, International Review of Economics and Finance 14, 469-486.

González-Rivera, G. (1998). Smooth Transition GARCH models. Studies in Nonlinear Dynamics and Econometrics, 3: 61-78.

Granger, C.W.J., Ding, Z., 1996, Varieties of long memory models, Journal of Econometrics 73, 61-78.

Granger, C.W.J., Hyung, N., 2004. Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns. Journal of Empirical Finance 11, 399.421.

Guidolin, M. and Timmermann, A. (2005). Economic implications of the bull and bear regimes in UK stock and bond returns. Economic Journal 115, 111-143.

Hagerud, G.E. (1997). “Specification tests for Asymmetric ARCH”. Working paper. Stockhold School of Economics.

Huang, K. (2011). Modelling volatility of S&P 500 index daily returns: A comparison between model based forecasts and implied volatility. PhD thesis.Department of Finance and Statistics, Hanken School of Economics, Vasa.

Hurst, H. (1951). Long term Storage capacity of Reservoirs. Transactions of the American Society of Civil Engineers 116, 770-779.

Jones, C.P., Walker, M.D. and Wilson, J.W. (2004). Analyzing stock market volatility using extreme day measures, Journal of Financial Research 27, 585-601.

Lambert, P. and Laurent, S. 2000. Modelling skewness dynamics in series of financial data. Discussion Paper, Institut de Statistique, Louvain la-Neuve.

Lambert, P. and Laurent, S. 2001. Modelling financial time series using GARCH-type models and a skewed Student density. Mimeo, Universite de Liege.

Laurent, S. (2007). Estimating and forecasting ARCH models using G@RCH 5. London: Timberlake Consultants Press.

Laurent, S. and Peters, J.P. (2006). G@RCH 4.2, Estimating and Forecasting ARCH Models. London: Timberlake Consultants Press.

Lo, A.W. (1991). Long term memory in stock market prices. Econometrica 59, 1279-1313.

Lobato, I.N., Savin, N.E., 1998, Real and spurious long memory properties of stock market data, Journal of Business and Economic Statistics 16, 261-268.

Maheu, J.M. and McCurdy, T.H. (2000). Identifying bull and bear markets in stock returns, Journal of Business and Economic Statistics 18, 100-112.

Newey, W.K. and K.D. (1987). A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. Econometrica, 55, 703-708.

Ohanissian, A., Russell, J.R., Tsay, R.S., 2008. True or spurious long memory? A new test. Journal of Business, Economic and Statistics 26, 161.175.

Pagan, A. and Sossounov, K.A. (2003). A simple framework for analysing bull and bear markets. Journal of Applied Econometrics 18, 23-46.

Perron, P., Qu, Z., 2010. Long-memory and level shifts in the volatility of stock market return indices. Journal of Business, Economics and Statistics 28, 275.290.

Phillips, P.C.B. and Shimotsu, K. (2004). Local Whittle estimation in nonstationary and unit root cases. Annals of Statistics, 32, 656-692.

Qu, Z., 2011. A test against spurious long memory. Journal of Business and Economic Statistics 29, 423.438.

Ramos, S.B.,Vermunt, J.K. and Dias, J.G. (2011). When markets fall down: are emerging markets all the same? International Journal of Finance and Economics 16, 324-338.

Robinson, P.M. (1994). Efficient tests of nonstationary hypotheses. Journal of the American Statistical Association 89, 1420-1437.

Robinson, P.M. (1995). Log-periodogram regression of time series with long range dependence. Annals of Statistics 23, 1048-1072.

Robinson, P.M. and M. Henry (1999), Long and short memory conditional Heteroskedasticity in estimating the memory in levels. Econometric Theory, 15, 299-336.

Shao, X., 2011. A simple test of changes in mean in the possible presence of long-range dependence. Journal of Time Series Analysis 32, 598.606.

Shimotsu, K. and Phillips, P.C.B. (2005). Exact local Whittle estimation of fractional integration. Annals of Statistics, 33, 1890-1933.

Sibbertsen, P., 2004, Long memory in volatilities of German stock returns, Empirical Economics 29, 477-488.

Sowell, F. (1992). Maximum likelihood estimation of stationary univariate fractionally Integrated time series models. Journal of Econometrics 53, 165-188.

Starica, C., Granger, C.W.J., 2005. Nonstationarities in stock returns. Review of Economics and Statistics 87, 503.522.

Teverovsky V., Taqqu, M.S. and Willinger, W. (1999). A critical look at Lo's modified R/S statistic. Journal of Statistical Planning and Inference 80, 211-227.

Tu, J. (2006). Are bull and bear markets economically important? Mimeo.

Velasco, C. and Robinson, P.M. (2000). Whittle pseudo maximum likelihood estimation for nonstationary time series. Journal of the American Statistical Association, 95, 1229-1243.

Wu, G. (2001). The determinants of asymmetric volatility. Review of Financial Studies, 14, 837 – 859.

Vilasuso, J. (2002). Forecasting exchange rate volatility. Economic Letters, 76, 59-64.

Xekalaki, E. and Degiannakis, S. (2010). ARCH models for financial applications. John Wiley & Sons. UK.

Figures and Tables

Table 1: Bull and bear phases in stock markets

1a) Europe			
Market Phases	FTSE	CAC40	DAX
1st (bear)	2002d13m03 –	2002d13m03 – 2002d22m10	2002d13m03 –
2nd (bull)	2002d02m10 –	2002d23m10 – 2007d23m11	2002d22m10 –
3rd (bear)	2007d16m11 –	2007d26m11 – 2009d16m03	2007d16m11 –
4th (bull)	2009d23m03 –	2009d17m03 – 2012d16m02	2009d31m03 –
1b) The US of America			
Market Phases	S & P	NASDAQ	DOW JONES
1st (bear)	2002d13m03 –	2002d13m03 – 2002d10m10	2002d13m03 –
2nd (bull)	2002d11m10 –	2002d11m10 – 2007d01m11	2002d03m10 –
3rd (bear)	2007d02m11 –	2007d02m11 – 2009d09m03	2007d30m11 –
4th (bull)	2009d11m03 –	2009d10m03 – 2012d16m02	2009d10m03 –
1c) Asia			
Market Phases	NIKKEI	HANG SENG	STI
1st (bear)	2002d13m03 –	2002d13m03 – 2002d01m10	2002d13m03 –
2nd (bull)	2002d03m10 –	2002d02m10 – 2007d09m11	2002d04m10 –
3rd (bear)	2007d06m11 –	2007d12m11 – 2009d02m03	2007d08m11 –
4th (bull)	2009d04m03 –	2009d03m03 – 2012d16m02	2009d04m03 –

Table 2: Descriptive measures of the stocks by market phase

2a) Europe						
	Market Phases	Mean	Median	St. dev.	Skewness	Kurtosis
FTSE	1st (bear)	-0.00108	-0.00068	0.00804	-0.1748	1.0023
	2nd (bull)	0.00020	0.00030	0.00416	0.0531	4.4595
	3rd (bear)	-0.00084	-0.00061	0.00957	0.0977	3.6931
	4th (bull)	0.00027	0.00035	0.00534	-0.0304	1.5327
CAC40	1st (bear)	-0.00143	-0.00184	0.01031	0.3239	1.1107

	2nd (bull)	0.00025	0.00038	0.00520	0.3127	4.3695
	3rd (bear)	-0.00096	-0.00026	0.01071	-0.6093	4.3080
	4th (bull)	0.00011	0.00026	0.00720	-0.3451	2.0891
DAX	1st (bear)	-0.00199	-0.00247	0.01059	0.2661	0.7211
	2nd (bull)	0.00039	0.00052	0.00571	0.0709	4.3318
	3rd (bear)	-0.00091	-0.00067	0.00930	-0.2819	6.3475
	4th (bull)	0.00033	0.00054	0.00681	-0.3005	2.2965
2b) The US of America						
	Market Phases	Mean	Median	St. dev.	Skewness	Kurtosis
S & P	1st (bear)	-0.00120	-0.00099	0.00764	0.5125	0.6983
	2nd (bull)	0.00024	0.00035	0.00372	0.1207	2.3332
	3rd (bear)	-0.00105	-0.00034	0.01001	-0.0671	3.8660
	4th (bull)	0.00036	0.00056	0.00539	-0.1706	3.4702
NASDAQ	1st (bear)	-0.00154	-0.00265	0.01003	0.1514	0.1562
	2nd (bull)	0.00032	0.00045	0.00474	0.1331	1.8868
	3rd (bear)	-0.00102	-0.00101	0.01036	-0.1833	2.9453
	4th (bull)	0.00044	0.00082	0.00560	-0.2085	1.6963
DOW JONES	1st (bear)	-0.00085	-0.00101	0.00542	0.0161	0.4129
	2nd (bull)	0.00020	0.00029	0.00403	0.3693	4.4006
	3rd (bear)	-0.00067	-0.00031	0.00932	0.1950	4.9217
	4th (bull)	0.00022	0.00032	0.00550	-0.1658	2.9644
2c) Asia						
	Market Phases	Mean	Median	St. dev.	Skewness	Kurtosis
NIKKEI	1st (bear)	-0.00097	-0.00121	0.00597	0.0118	-0.3286
	2nd (bull)	0.00021	0.00025	0.00499	-0.1700	0.5212
	3rd (bear)	-0.00070	-0.00080	0.01014	-0.0755	4.2425
	4th (bull)	0.00000	0.00004	0.00521	-0.4444	2.7010
HANG SENG	1st (bear)	-0.00060	-0.00106	0.00544	0.2031	-0.2599
	2nd (bull)	0.00040	0.00051	0.00501	-0.0992	2.0334
	3rd (bear)	-0.00101	-0.00072	0.01386	0.3047	2.9940
	4th (bull)	0.00018	0.00016	0.00626	-0.0570	1.7241
STI	1st (bear)	-0.00092	-0.00118	0.00488	0.2790	1.0457
	2nd (bull)	0.00036	0.00042	0.00446	-0.3485	3.8367
	3rd (bear)	-0.00124	-0.00122	0.00914	-0.0208	2.3019
	4th (bull)	0.00036	0.00021	0.00500	0.3357	2.8904

Table 3: Estimates of the fractional differencing parameters in the stock returns with the parametric approach

Europe		No regressors	An intercept	A linear trend
FTSE	1ST Bear	**-0.087 (-0.19, 0.06)**	**-0.092 (-0.21, 0.06)**	**-0.104 (-0.22, 0.06)**
	2nd Bull	**-0.116 (-0.16, 0.07)**	-0.116 (-0.16, -0.07)	-0.116 (-0.16, -0.07)
	3rd Bear	-0.104 (-0.18, -0.01)	-0.105 (-0.18, -0.01)	-0.109 (-0.18, -0.01)
	4rd Bull	-0.031 (-0.08, -0.02)	-0.031 (-0.08, -0.02)	-0.042 (-0.09, -0.02)
		No regressors	An intercept	A linear trend
CAC40	1ST Bear	-0.144 (-0.23, -0.01)	-0.157 (-0.26, -0.01)	-0.177 (-0.29, -0.02)
	2nd Bull	**-0.099 (-0.14, 0.06)**	**-0.099 (-0.14, 0.06)**	**-0.099 (-0.14, 0.06)**
	3rd Bear	-0.194 (-0.25, -0.13)	-0.202 (-0.27, -0.13)	-0.213 (-0.28, -0.14)
	4rd Bull	-0.056 (-0.10, -0.01)	-0.056 (-0.10, -0.01)	-0.063 (-0.11, -0.01)
		No regressors	An intercept	A linear trend
DAX	1ST Bear	**-0.098 (-0.19, 0.02)**	**-0.112 (-0.21, 0.03)**	**-0.169 (-0.29, -0.00)**
	2nd Bull	**-0.037 (-0.07, 0.00)**	**-0.037 (-0.07, 0.00)**	**-0.037 (-0.07, 0.00)**
	3rd Bear	**-0.031 (-0.11, 0.06)**	**-0.032 (-0.11, 0.06)**	**-0.035 (-0.12, 0.06)**
	4rd Bull	**-0.035 (-0.08, 0.02)**	**-0.034 (-0.07, 0.02)**	**-0.041 (-0.09, 0.01)**
U.S.A.		No regressors	An intercept	A linear trend
S&P	1ST Bear	**-0.082 (-0.19, 0.08)**	**-0.083 (-0.20, 0.08)**	**-0.082 (-0.20, 0.08)**
	2nd Bull	-0.067 (-0.10, -0.02)	-0.065 (-0.10, -0.02)	-0.066 (-0.10, -0.02)
	3rd Bear	-0.122 (-0.18, -0.05)	-0.127 (-0.19, -0.05)	-0.152 (-0.22, -0.07)
	4rd Bull	**-0.037 (-0.08, 0.01)**	**-0.035 (-0.08, 0.01)**	**-0.041 (-0.09, 0.01)**
		No regressors	An intercept	A linear trend
NASDAQ	1ST Bear	-0.237 (-0.33, -0.11)	-0.250 (-0.36, -0.11)	-0.249 (-0.36, -0.11)
	2nd Bull	**-0.022 (-0.06, 0.01)**	**-0.022 (-0.06, 0.01)**	**-0.023 (-0.06, 0.01)**
	3rd Bear	-0.098 (-0.16, -0.02)	-0.099 (-0.16, -0.02)	-0.107 (-0.17, -0.02)
	4rd Bull	**0.019 (-0.03, 0.08)**	**0.019 (-0.03, 0.08)**	**0.014 (-0.04, 0.08)**
		No regressors	An intercept	A linear trend
DOW JONES	1ST Bear	**0.043 (-0.04, 0.16)**	**0.044 (-0.04, 0.16)**	**-0.008 (-0.11, 0.12)**
	2nd Bull	-0.049 (-0.09, -0.01)	-0.049 (-0.09, -0.01)	-0.049 (-0.09, -0.01)
	3rd Bear	-0.182 (-0.24, -0.11)	-0.191 (-0.23, -0.11)	-0.213 (-0.28, -0.13)
	4rd Bull	-0.059 (-0.09, -0.01)	-0.059 (-0.10, -0.01)	-0.059 (-0.10, -0.01)
Asia		No regressors	An intercept	A linear trend
Nikkei	1ST Bear	**-0.029 (-0.14, 0.12)**	**-0.030 (-0.14, 0.12)**	**-0.049 (-0.17, 0.11)**
	2nd Bull	**0.022 (-0.02, 0.06)**	**0.022 (-0.02, 0.06)**	**0.021 (-0.02, 0.06)**
	3rd Bear	**0.046 (-0.04, 0.15)**	**0.046 (-0.04, 0.15)**	**0.046 (-0.04, 0.15)**
	4rd Bull	**0.051 (0.00, 0.11)**	**0.051 (0.00, 0.11)**	**0.050 (0.00, 0.11)**
		No regressors	An intercept	A linear trend
HANG SENG	1ST Bear	-0.056 (-0.14, -0.06)	-0.059 (-0.15, -0.06)	-0.110 (-0.22, -0.04)
	2nd Bull	-0.038 (-0.07, -0.01)	-0.038 (-0.08, -0.01)	-0.043 (-0.08, -0.01)
	3rd Bear	-0.125 (-0.19, -0.04)	-0.125 (-0.20, -0.04)	-0.124 (-0.19, -0.04)
	4rd Bull	**-0.021 (-0.03, 0.08)**	**-0.021 (-0.03, 0.08)**	**-0.012 (-0.04, 0.07)**
		No regressors	An intercept	A linear trend
STI	1ST Bear	**- 0.113 (-0.20, 0.01)**	**- 0.111 (-0.23, 0.01)**	**- 0.134 (-0.25, 0.01)**
	2nd Bull	- 0.061 (-0.09, -0.02)	- 0.062 (-0.09, -0.02)	- 0.063 (-0.09, -0.02)
	3rd Bear	**- 0.038 (-0.10, 0.04)**	**- 0.039 (-0.10, 0.04)**	**- 0.045 (-0.11, 0.04)**
	4rd Bull	**0.042 (0.00, 0.09)**	**0.041 (0.00, 0.08)**	**0.027 (-0.02, 0.08)**

In bold, statistical evidence of I(0) behavior.

Table 4: Estimates of the fractional differencing parameters in the stock returns with the semiparametric approach

		FTSE	CAC40	DAX
Europe	1ST Bear	0.088	**-0.176**	-0.024
	2nd Bull	-0.091	-0.064	0.014
	3rd Bear	-0.067	**-0.116**	**-0.237**
	4rd Bull	-0.046	-0.065	0.093
		S&P	NASDAQ	DOW JONES
USA	1ST Bear	-0.092	**-0.298**	0.273
	2nd Bull	-0.046	0.096	**-0.211**
	3rd Bear	0.058	0.087	-0.041
	4rd Bull	-0.031	-0.063	**-0.132**
		NIKKEI	HANG SENG	STI
Asia	1ST Bear	0.017	0.177	**-0.129**
	2nd Bull	-0.019	-0.049	-0.060
	3rd Bear	0.128	**-0.121**	0.065
	4rd Bull	**-0.101**	-0.034	0.120

In bold, significant evidence of mean reversion in the log prices.

Table 5: Estimates of the fractional differencing parameters in the stock returns with the nonparametric approach

		FTSE	CAC40	DAX
Europe	1ST Bear	**-0.008**	**-0.031**	**-0.034**
	2nd Bull	**-0.027**	**-0.029**	**0.007**
	3rd Bear	**-0.045**	**-0.052**	**-0.012**
	4rd Bull	**0.018**	**0.010**	**0.021**
		S&P	NASDAQ	DOW JONES
USA	1ST Bear	**0.026**	**-0.038**	**0.076**
	2nd Bull	**-0.010**	**0.042**	**-0.027**
	3rd Bear	**-0.011**	**0.001**	**-0.021**
	4rd Bull	**0.0237**	**0.034**	**0.013**
		NIKKEI	HANG SENG	STI
Asia	1ST Bear	**-0.011**	**0.014**	**-0.057**
	2nd Bull	**0.020**	**0.008**	**-0.021**
	3rd Bear	**0.042**	**-0.001**	**0.019**
	4rd Bull	**-0.003**	**0.033**	**0.071**

In bold, statistical evidence of I(0) behavior.

Table6: Estimates of the fractional differencing parameters in the squared returns with the parametric approach

Europe		No regressors	An intercept	A linear trend
FTSE	1ST Bear	**0.153 (0.10, 0.21)**	**0.169 (0.12, 0.24)**	**0.098 (0.04, 0.17)**
	2nd Bull	**0.255 (0.23, 0.28)**	**0.238 (0.21, 0.27)**	**0.241 (0.21, 0.28)**
	3rd Bear	**0.178 (0.13, 0.23)**	**0.184 (0.14, 0.24)**	**0.171 (0.13, 0.24)**
	4rd Bull	**0.149 (0.11, 0.19)**	**0.139 (0.11, 0.18)**	**0.143 (0.11, 0.18)**
		No regressors	An intercept	A linear trend
CAC40	1ST Bear	**0.185 (0.12, 0.27)**	**0.205 (0.14, 0.29)**	**0.132 (0.05, 0.23)**
	2nd Bull	**0.174 (0.15, 0.20)**	**0.157 (0.14, 0.18)**	**0.146 (0.13, 0.17)**
	3rd Bear	**0.219 (0.17, 0.28)**	**0.225 (0.17, 0.28)**	**0.212 (0.16, 0.28)**
	4rd Bull	**0.128 (0.10, 0.16)**	**0.128 (0.10, 0.16)**	**0.126 (0.10, 0.16)**
		No regressors	An intercept	A linear trend
DAX	1ST Bear	**0.164 (0.11, 0.24)**	**0.187 (0.13, 0.27)**	**0.094 (0.02, 0.19)**
	2nd Bull	**0.211 (0.19, 0.23)**	**0.189 (0.17, 0.21)**	**0.174 (0.15, 0.20)**
	3rd Bear	**0.147 (0.11, 0.19)**	**0.152 (0.11, 0.19)**	**0.135 (0.09, 0.19)**
	4rd Bull	**0.150 (0.12, 0.17)**	**0.151 (0.12, 0.18)**	**0.145 (0.12, 0.17)**
U.S.A.		No regressors	An intercept	A linear trend
S&P	1ST Bear	**0.114 (0.06, 0.19)**	**0.131 (0.07, 0.21)**	**0.076 (0.00, 0.17)**
	2nd Bull	**0.175 (0.15, 0.20)**	**0.156 (0.13, 0.18)**	**0.154 (0.13, 0.18)**
	3rd Bear	**0.167 (0.13, 0.21)**	**0.175 (0.14, 0.22)**	**0.149 (0.11, 0.20)**
	4rd Bull	**0.187 (0.15, 0.22)**	**0.179 (0.15, 0.22)**	**0.187 (0.15, 0.22)**
		No regressors	An intercept	A linear trend
NASDAQ	1ST Bear	**0.134 (0.05, 0.26)**	**0.151 (0.06, 0.28)**	**0.145 (0.06, 0.27)**
	2nd Bull	**0.145 (0.12, 0.17)**	**0.124 (0.10, 0.15)**	**0.103 (0.08, 0.13)**
	3rd Bear	**0.293 (0.24, 0.35)**	**0.298 (0.25, 0.36)**	**0.291 (0.24, 0.35)**
	4rd Bull	**0.172 (0.14, 0.21)**	**0.163 (0.13, 0.20)**	**0.165 (0.14, 0.20)**
		No regressors	An intercept	A linear trend
DOW JONES	1ST Bear	**0.124 (0.03, 0.24)**	**0.131 (0.04, 0.25)**	**0.098 (0.00, 0.22)**
	2nd Bull	**0.186 (0.17, 0.21)**	**0.168 (0.15, 0.19)**	**0.160 (0.14, 0.18)**
	3rd Bear	**0.168 (0.13, 0.21)**	**0.177 (0.14, 0.22)**	**0.118 (0.08, 0.17)**
	4rd Bull	**0.163 (0.13, 0.19)**	**0.153 (0.13, 0.18)**	**0.149 (0.12, 0.18)**
Asia		No regressors	An intercept	A linear trend
Nikkei	1ST Bear	-0.054 (-0.11, 0.04)	-0.063 (-0.14, 0.04)	-0.083 (-0.17, 0.02)
	2nd Bull	**0.152 (0.12, 0.18)**	**0.134 (0.10, 0.18)**	**0.128 (0.10, 0.17)**
	3rd Bear	**0.247 (0.20, 0.30)**	**0.249 (0.21, 0.30)**	**0.248 (0.21, 0.30)**
	4rd Bull	**0.246 (0.19, 0.31)**	**0.244 (0.19, 0.31)**	**0.244 (0.19, 0.30)**
		No regressors	An intercept	A linear trend
HANG SENG	1ST Bear	0.021 (-0.05, 0.11)	0.025 (-0.06, 0.13)	-0.001 (-0.10, 0.10)
	2nd Bull	**0.157 (0.13, 0.18)**	**0.155 (0.13, 0.17)**	**0.149 (0.13, 0.18)**
	3rd Bear	**0.220 (0.17, 0.28)**	**0.222 (0.18, 0.29)**	**0.219 (0.17, 0.28)**
	4rd Bull	**0.134 (0.11, 0.17)**	**0.123 (0.10, 0.16)**	**0.124 (0.10, 0.17)**
		No regressors	An intercept	A linear trend
STI	1ST Bear	0.062 (-0.01, 0.17)	0.073 (-0.01, 0.19)	0.042 (-0.06, 0.18)
	2nd Bull	**0.152 (0.13, 0.18)**	**0.149 (0.14, 0.18)**	**0.148 (0.14, 0.17)**
	3rd Bear	**0.177 (0.13, 0.23)**	**0.182 (0.14, 0.24)**	**0.176 (0.13, 0.23)**
	4rd Bull	**0.162 (0.13, 0.19)**	**0.146 (0.12, 0.17)**	**0.145 (0.12, 0.18)**

In bold, significant evidence of long memory behavior (d > 0)

Table7: Estimates of the fractional differencing parameters in the squaredreturnswith the semiparametric approach

		FTSE	CAC40	DAX
Europe	1ST Bear	**0.385**	**0.413**	**0.421**
	2nd Bull	**0.414**	**0.445**	**0.462**
	3rd Bear	**0.452**	**0.430**	**0.437**
	4rd Bull	**0.427**	**0.354**	**0.409**
		S&P	NASDAQ	DOW JONES
USA	1ST Bear	**0.453**	**0.364**	**0.144**
	2nd Bull	**0.406**	**0.477**	**0.372**
	3rd Bear	**0.488**	**0.420**	**0.491**
	4rd Bull	**0.357**	**0.493**	**0.493**
		NIKKEI	HANG SENG	STI
Asia	1ST Bear	**0.273**	**0.248**	**0.074**
	2nd Bull	**0.434**	**0.433**	**0.423**
	3rd Bear	**0.453**	**0.472**	**0.479**
	4rd Bull	**0.008**	**0.480**	**0.449**

In bold, significant evidence of long memory behavior (d > 0)

Table 8: Estimates of the fractional differencing parameters in the squared returnswith the nonparametric approach

		FTSE	CAC40	DAX
Europe	1ST Bear	**0.198**	**0.193**	**0.212**
	2nd Bull	**0.236**	**0.233**	**0.256**
	3rd Bear	**0.200**	**0.175**	**0.175**
	4rd Bull	**0.156**	**0.186**	**0.213**
		S&P	NASDAQ	DOW JONES
USA	1ST Bear	**0.155**	**0.1488**	**0.086**
	2nd Bull	**0.225**	**0.224**	**0.240**
	3rd Bear	**0.219**	**0.213**	**0.215**
	4rd Bull	**0.176**	**0.154**	**0.205**
		NIKKEI	HANG SENG	STI
Asia	1ST Bear	**0.065**	**0.061**	**0.095**
	2nd Bull	**0.169**	**0.204**	**0.214**
	3rd Bear	**0.194**	**0.199**	**0.168**
	4rd Bull	**0.095**	**0.183**	**0.203**

In bold, significant evidence of long memory behavior (d > 0)

Table 9: Estimates of GJR models for European stock markets

Market Phases	Estimates	FTSE	CAC40	DAX
1st	$\hat{\phi}_0$	-0.001058***	-0.001369***	-0.001803***
	$\hat{\phi}_1$	-0.073674***	-0.127517***	-0.004170
	$\hat{w}$	0.033092***	0.021840***	0.016510
	$\hat{\alpha}_1$	-0.623398***	-0.098339***	-0.038731
	$\hat{\beta}_1$	0.959584***	0.875256***	0.868255***
	$\hat{\gamma}_1$	1.552118***	0.497194***	0.393643***
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.336186	0.776917	0.829524
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.888304	1.274111	1.223167
2nd	$\hat{\phi}_0$	0.000597	0.001887***	0.000375***
	$\hat{\phi}_1$	-0.094696***	-0.094750***	-0.034481
	$\hat{w}$	0.000886	0.000842***	-0.000238
	$\hat{\alpha}_1$	0.015039***	0.007372***	0.034842***
	$\hat{\beta}_1$	0.890015***	0.912241***	0.920775***
	$\hat{\gamma}_1$	0.203756***	0.185482***	0.090893***
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.905054	0.919613	0.955617
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.108810	1.105095	1.046510
3rd	$\hat{\phi}_0$	-0.000746***	-0.000897***	-0.000678***
	$\hat{\phi}_1$	-0.086548	-0.181226***	-0.006900
	$\hat{w}$	0.017692***	0.024370***	0.016489
	$\hat{\alpha}_1$	-0.053381***	-0.038820	-0.022130
	$\hat{\beta}_1$	0.912363***	0.884333***	0.886783***
	$\hat{\gamma}_1$	0.251017***	0.260986***	0.236617***
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.858982	0.845513	0.864653
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.109999	1.106499	1.101270
4th	$\hat{\phi}_0$	0.000100	0.000044	0.000195
	$\hat{\phi}_1$	-0.011890	-0.043985	-0.038521
	$\hat{w}$	0.007396***	0.007493***	0.004067
	$\hat{\alpha}_1$	-0.007799	-0.014726	-0.019657
	$\hat{\beta}_1$	0.896308***	0.916246***	0.927971
	$\hat{\gamma}_1$	0.175656	0.161236***	0.163038
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.888509	0.901520	0.908314
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.064165	1.062756	1.071352

Table 10: Estimates of GJR models for the Asianstock markets

Market Phases	Estimates	S & P	NASDAQ	DOW JONES
1^{st}	$\hat{\phi}_0$	-0.001486***	-0.001607***	-0.000671
	$\hat{\phi}_1$	-0.069103	-0.080759***	-0.069256
	$\hat{w}$	0.014623	0.356163***	0.001662
	$\hat{\alpha}_1$	-0.035401	-0.145901***	-0.062511
	$\hat{\beta}_1$	0.888707***	0.549502***	0.984345***
	$\hat{\gamma}_1$	0.318655***	0.512620***	0.200349
	$\hat{\alpha}_1+\hat{\beta}_1$	0.853306	0.403601	0.921834
	$\hat{\alpha}_1+\hat{\beta}_1+\hat{\gamma}_1$	1.171961	0.916221	1.122183
2^{nd}	$\hat{\phi}_0$	0.001098	0.000244	0.000928
	$\hat{\phi}_1$	-0.087473***	-0.028770	-0.069248***
	$\hat{w}$	0.002781	-0.000784	-0.001656***
	$\hat{\alpha}_1$	0.062081	0.034105	0.016596***
	$\hat{\beta}_1$	0.920322***	0.960217	0.940746***
	$\hat{\gamma}_1$	0.174457***	0.016557***	0.124890***
	$\hat{\alpha}_1+\hat{\beta}_1$	0.982403	0.994322	0.957342
	$\hat{\alpha}_1+\hat{\beta}_1+\hat{\gamma}_1$	1.015686	1.010879	1.082232
3^{rd}	$\hat{\phi}_0$	-0.000780***	-0.000842***	-0.0004705***
	$\hat{\phi}_1$	-0.065945***	-0.071417	-0.091288***
	$\hat{w}$	0.017847***	0.029356***	0.109546***
	$\hat{\alpha}_1$	-0.067045***	-0.031306	-1.021728***
	$\hat{\beta}_1$	0.903325***	0.871010***	0.946778***
	$\hat{\gamma}_1$	0.272390***	0.257248***	2.727204***
	$\hat{\alpha}_1+\hat{\beta}_1$	0.83628	0.839704	-0.07495
	$\hat{\alpha}_1+\hat{\beta}_1+\hat{\gamma}_1$	1.108670	1.096952	2.652254
4^{th}	$\hat{\phi}_0$	0.000211	0.000448	0.000145
	$\hat{\phi}_1$	-0.003928	-0.017315	-0.062625***
	$\hat{w}$	0.004773***	0.004543	0.003570***
	$\hat{\alpha}_1$	0.038565***	0.048532	-0.024980***
	$\hat{\beta}_1$	0.875419***	0.886313***	0.917998***
	$\hat{\gamma}_1$	0.164054***	0.091257	0.215603***
	$\hat{\alpha}_1+\hat{\beta}_1$	0.913984	0.934845	0.893018
	$\hat{\alpha}_1+\hat{\beta}_1+\hat{\gamma}_1$	1.078038	1.026102	1.108621

Table 11: Estimates of GJR models for the American stock markets

Market Phases	Estimates	NIKKEI	HANG SENG	STI
1st	$\hat{\phi}_0$	-0.000776***	-	-
	$\hat{\phi}_1$	-0.137661***	-	-
	$\hat{w}$	0.291035***	-	-
	$\hat{\alpha}_1$	0.405267***	-	-
	$\hat{\beta}_1$	0.510725***	-	-
	$\hat{\gamma}_1$	0.167728***	-	-
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.915992	-	-
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.083720	-	-
2nd	$\hat{\phi}_0$	0.000241***	0.000350***	0.000299***
	$\hat{\phi}_1$	0.040357***	-0.052561***	-0.031422
	$\hat{w}$	0.009162***	0.001311	0.001186***
	$\hat{\alpha}_1$	0.039452***	0.043272***	0.030944***
	$\hat{\beta}_1$	0.868178***	0.952915***	0.933046***
	$\hat{\gamma}_1$	0.111486***	0.001031	0.072921***
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.907630	0.996187	0.963990
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.019116	0.997218	1.036911
3rd	$\hat{\phi}_0$	-0.000724	-0.001146***	-0.001281***
	$\hat{\phi}_1$	0.169383***	-0.114982***	-0.058984
	$\hat{w}$	0.026704	0.026021	0.010150
	$\hat{\alpha}_1$	0.031663***	0.002172	-0.030523
	$\hat{\beta}_1$	0.845261***	0.897389***	0.943278***
	$\hat{\gamma}_1$	0.182342	0.180050***	0.163706***
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.876924	0.899561	0.912755
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	1.059266	1.079611	1.076461
4th	$\hat{\phi}_0$	0.000032	0.000077	0.000187
	$\hat{\phi}_1$	0.035263	0.043655	-0.007869
	$\hat{w}$	0.032992	0.004469***	-0.001511***
	$\hat{\alpha}_1$	-0.006482	0.029001***	0.071187***
	$\hat{\beta}_1$	0.800590***	0.944278***	0.897265***
	$\hat{\gamma}_1$	0.150723	0.027175	0.073410***
	$\hat{\alpha}_1 + \hat{\beta}_1$	0.794108	0.973279	0.968452
	$\hat{\alpha}_1 + \hat{\beta}_1 + \hat{\gamma}_1$	0.944831	1.000454	1.041862