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Competitive Pressure and Innovation in Vertically Differentiates Markets

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Abstract

This paper analyzes the innovation incentives for a firm that produces a vertically differentiated product, in the presence of a competitive fringe that produces a lower-quality product. I find that the relationship between innovation incentives and competition, measured by the difference in quality levels may exhibit an inverted-U-shaped pattern.

1 Introduction

Ever since Arrow's seminal contribution (see Arrow (1962)), the effect of the degree of competition on the incentives to introduce new products and processes has attracted the interest of researchers in the field of Economics, see Vives (2008) for a recent summary of the main findings of the literature on the relationship between competitive pressure and innovation. In this paper, I develop a theoretical model where a firm producing a higher-quality good than a competitive fringe may invest resources to further increase the quality of its product. I examine the incentives of that firm to engage in costly quality upgrading as a function of the difference in quality relative to the competitive fringe to find that there are parameter values such that the relationship between competitive pressure and incentives to innovate has an inverted-U shape, as found in Aghion et al. (2005). This model may be used to analyze the incentives that firms in the formal sector of developing countries have to innovate, in the presence of a competitive informal sector. In fact, firms in the informal sector of developing countries typically produce lower-quality varieties of the products produced by firms in the formal sector, as in Banerji and Jain (2007).

The theoretical model is based on that in Shaked and Sutton (1982), which develops a model of vertical differentiation that has been extensively used in the Industrial Organization literature. I analyze the high-quality firm's incentives to undertake an investment conducive towards introducing an improvement in product quality, as a function of the competitive pressure by producers of a lower-quality product. This question is related to those addressed in Greenstein and Ramey (1998), or Chen and Schwartz (2013). In the

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model, competitive pressure by the informal sector is measured as the difference in quality between the two products, and thus, a more intense competitive pressure is represented by a smaller difference in quality. I find that there are parameter values such that the relationship between competitive pressure and incentives to innovate has an inverted-U shape.

The rest of the paper is organized as follows: Section 2 develops the model and presents the basic results. Section 3 discusses the different possible relationships between competitive pressure and innovation. Finally, Section 4 presents some concluding comments.

2 The model

The model analyzes a setting in which a firm is the only producer of a high-quality product, whereas a lower-quality version of the product is produced by a number of firms which compete in prices. Consumers have heterogeneous willingness to pay for quality. The foundations of this model may be found in Shaked and Sutton (1982) or Shaked and Sutton (1983), see Tirole (1989) for a simplified version of these models. In this paper, competitive pressure is measured by the difference in quality levels between high- and low-quality products.

Let firm 1 produce a product with a quality level s_1 . Firm 1 faces competition against a competitive fringe, composed of at least two firms that produce with quality level $s_2 < s_1$. Marginal costs are also asymmetric. Specifically, firm 1's marginal cost, c_1 is at least as high as that of the firms that produce the low-quality product, c_2 , hence $c_2 \leq c_1$. On the demand side, there is a continuum of consumers, with total mass m . The preferences of the consumers are characterized by the following utility function:

$$u(\theta) = \theta s - p \quad (1)$$

where $\theta \sim U[\underline{\theta}, \bar{\theta}]$ and p is the price paid by the product that each consumer consume. Consumers purchase at most one unit of one of the two products. This way, these two products are vertically-differentiated imperfect substitutes. Given the preferences of the consumers and the distribution of their willingness to pay, the expressions for the demand for the high- and low-quality goods (assuming that the other type of good is not offered) are

$$p_i = s_i \left[\bar{\theta} - \frac{\bar{\theta} - \underline{\theta}}{m} q_i \right] \quad (2)$$

where $i = 1, 2$ denotes the high- and low-quality products, respectively.

Given quality levels s_1 , s_2 , and marginal costs c_1 and c_2 , depending on the values of these parameters, we could come up with two market outcomes, namely whether or not firm 1 faces competition against producers of the low-quality product. First, if the difference in quality levels, $s_1 - s_2$ is big enough, then there will be no active firms in the low-quality segment. This occurs when the market share of a competitive low-quality segment is zero even if the high-quality firm charges the monopoly price. In this case, a high-quality monopolist that faces demand given by (2) would optimally set

$$q^M = \frac{m(s_1 \bar{\theta} - c_1)}{2s_1(\bar{\theta} - \underline{\theta})} \text{ and } p^M = \frac{\bar{\theta}s_1 + c_1}{2}$$

yielding profits

$$\pi_1^M = \frac{m (s_1 \bar{\theta} - c_1)^2}{4s_1 (\bar{\theta} - \underline{\theta})}$$

and it is easy to see that

$$\frac{\partial \pi_1^M}{\partial s_1} = \frac{m}{4 (\bar{\theta} - \underline{\theta})} \left[\bar{\theta}^2 - \left(\frac{c_1}{s_1} \right)^2 \right]$$

which is positive whenever $\bar{\theta}s_1 > c_1$ and, of course, does not depend on s_2 . Firm 1 will not face competition against producers of the low-quality product as long as the willingness to pay for the low-quality product by the last consumer served by firm 1 if it charges the monopoly price be below the cost of the low-quality producer. This is the case whenever

$$s_1 \geq \tilde{s}_1(s_2) = \frac{s_2 c_1}{2c_2 - \bar{\theta}s_2} \quad (3)$$

which requires that $2c_2 \geq \bar{\theta}s_2$. That is, in order for low-quality producers not to have a positive market share, their costs must be high enough relative to their quality levels. Notice that this condition also defines a minimum quality level for the high-quality producer such that it eliminates competition from the low-quality producers, $\tilde{s}_1(s_2)$. As one would easily anticipate, given that $2c_2 > \bar{\theta}s_2$, $\frac{\partial \tilde{s}_1}{\partial s_2} > 0$, that is, an increase in the low-quality producers' level of quality increases the minimum level of quality of the high-quality producer such that the market share of the competitive fringe is zero.

The other market configuration arises if the difference in quality levels is not high enough, given marginal costs c_1 and c_2 . In this case, the high-quality product will co-exist with the low-quality product and the producers of the two vertically-differentiated products will compete in prices. Let p_1 and p_2 be the prices posted by the high- and low-quality firms, respectively. Then, there will be an indifferent consumer, with willingness to pay $\tilde{\theta}$ that is characterized by

$$\tilde{\theta}s_1 - p_1 = \tilde{\theta}s_2 - p_2$$

which implies that

$$\tilde{\theta} = \frac{p_1 - p_2}{s_1 - s_2} \quad (4)$$

Having identified the indifferent consumer, since consumers' willingness to pay are distributed according to $U [\underline{\theta}, \bar{\theta}]$, the expression for the quantity demanded of the high-quality product is:

$$q_1(p_1, p_2, s_1, s_2) = \frac{m}{\bar{\theta} - \underline{\theta}} \left[\bar{\theta} - \frac{p_1 - p_2}{s_1 - s_2} \right] \quad (5)$$

Therefore, firm 1's reaction function is given by

$$p_1 = \frac{\bar{\theta}(s_1 - s_2) + p_2 + c_1}{2} \quad (6)$$

Now, if the firms that produce the low-quality product are competitive, then it will be the case that $p_2 = c_2$ since there is no product differentiation among producers of the low-quality product. Hence, firm 1's optimal price is

$$p_1 = \frac{\bar{\theta}(s_1 - s_2) + c_1 + c_2}{2}$$

with the quantity sold by firm 1 being

$$q_1 = \frac{m}{(\bar{\theta} - \underline{\theta})(s_1 - s_2)} \frac{\bar{\theta}(s_1 - s_2) + c_2 - c_1}{2}$$

and yielding profits

$$\pi_1(s_2) = \frac{m [\bar{\theta}(s_1 - s_2) + c_2 - c_1]^2}{4(\bar{\theta} - \underline{\theta})(s_1 - s_2)}$$

Now, given this expression for firm 1's profits, we can easily see that

$$\frac{\partial \pi_1(s_2)}{\partial s_1} = \frac{m}{4(\bar{\theta} - \underline{\theta})} \left[\bar{\theta}^2 - \left(\frac{c_1 - c_2}{s_1 - s_2} \right)^2 \right]$$

which is positive as long as $\bar{\theta}(s_1 - s_2) > c_1 - c_2$, which is necessary in order for firm 1 to have a positive market share. Additionally,

$$\frac{\partial^2 \pi_1(s_2)}{\partial s_1 \partial s_2} = -\frac{m(c_1 - c_2)^2}{2(\bar{\theta} - \underline{\theta})(s_1 - s_2)^3} < 0 \quad (7)$$

In the model, firm 1 has the ability to invest in raising the quality level of its product. Specifically, if we refer to the initial quality level of firm 1 as s_1^0 , by choosing an investment level e , firm 1 increases the level of quality of its own product by $g(e)$, at a cost given by αe . The function $g(\cdot)$ is such that $g'(\cdot) > 0$ and $g''(\cdot) \leq 0$, that is, subject to decreasing returns. Notice also that the model assumes that the increase in quality $g(e)$ is independent of the initial level of quality. Therefore, following an investment $e \geq 0$, the quality of the product that firm 1 produces is raised to $s_1^0 + g(e)$. Also assume that the $g(\cdot)$ function satisfies

$$\lim_{e \rightarrow 0} g'(e) = \infty \text{ and } \lim_{e \rightarrow \infty} g'(e) = 0 \quad (8)$$

and that $g(\cdot)$ is such that $\frac{\partial \pi_1}{\partial s_1} \frac{\partial s_1}{\partial e}$ is a decreasing function of e . The fundamental issue this paper analyzes is how the level of investment in innovation e varies with $s_1^0 - s_2$. Given these preliminaries, firm 1's problem reads:

$$\max_{e \geq 0} \pi_1(s_1^0 + g(e), s_2) - \alpha e \quad (9)$$

where $\pi_1(\cdot)$ are firm 1's gross profits, i.e. prior to subtracting the cost of the investment. The first-order condition of this problem is given by

$$\frac{\partial \pi_1}{\partial s_1} \frac{\partial s_1}{\partial e} = \alpha \quad (10)$$

if the high-quality firm optimally chooses $e^* > 0$, which must be the case if (8) holds.

Notice that the expression for the first-order condition includes the product of two partial derivatives. On the one hand, the function $\frac{\partial s_1}{\partial e}$ is determined by the $g(\cdot)$ function and is therefore independent of the degree of competition against the informal sector. On the other hand, the expression $\frac{\partial \pi_1}{\partial s_1}$ describes how the high-quality firm's profits increase with its own quality level. The functional form of this expression which differs depending on whether or not the market share of the low-quality product is positive.

Therefore, firm 1's incentives to invest in quality upgrading will depend on whether it faces competition against producers of low-quality products, which depends on parameter values. The following section analyzes these different cases.

3 Competitive pressure and incentives to innovate

As pointed out in the previous section, firm 1's incentives to upgrade the quality of its product depend on whether it faces competition against producers of low-quality products. First, notice that a necessary condition for low-quality producers to exist is that $\bar{\theta}s_2 > c_2$, otherwise, firm 1 behaves as an undisputed monopoly. On the other hand, recall that if $2c_2 - \bar{\theta}s_2 \leq 0$ then the low-quality producer's market share will always be positive. In the latter case, by (7), the relationship between competitive pressure and incentives to innovate is negative, that is $\frac{\partial e^*}{\partial s_2} < 0$.

Now focus on $s_2 \in \left[\frac{c_2}{\bar{\theta}}, \frac{2c_2}{\bar{\theta}}\right]$. Then, a finite threshold value $\tilde{s}_1(s_2)$ exists. Notice that for $s_1 < \tilde{s}_1(s_2)$ firm 2 shares the market with producers of low-quality products, whereas if $s_1 \geq \tilde{s}_1(s_2)$ then firm 1 is the only active producer in the market. Further notice that the expression for the derivative of the profits of firm 1 with respect to its own level of quality exhibits a discontinuity precisely at \tilde{s}_1 . To see this, notice that whenever $\frac{c_1}{s_1} > \frac{c_1 - c_2}{s_1 - s_2}$ then

$$\frac{\partial \pi_1(s_2)}{\partial s_1} > \frac{\partial \pi_1^M}{\partial s_1} \quad (11)$$

and when $s_1 = \tilde{s}_1(s_2)$, the condition becomes $\bar{\theta}s_2 > c_2$, which must necessarily hold. Therefore, at $\tilde{s}_1(s_2)$ there is a discontinuity in the expression of the derivative of firm 1's profits with respect to its level of quality, as seen in (11). Furthermore, the partial derivative at $\tilde{s}_1(s_2)$ is greater if low-quality producers have a positive market share.

Additionally, recall that, provided that $\bar{\theta}s_2 < 2c_2$ we know that $\frac{\partial \tilde{s}_1(s_2)}{\partial s_2} > 0$. Let e^M be the value of e such that

$$\frac{\partial \pi_1^M}{\partial s_1} \frac{\partial s_1}{\partial e} = \alpha.$$

that is, the effort level a monopolist firm 1 would choose absent competitive pressure from producers of low-quality products.

First, if for every $s_2 \in \left[\frac{c_2}{\bar{\theta}}, \frac{2c_2}{\bar{\theta}}\right]$, $s_1^0 + g(e^M) < \tilde{s}_1(s_2)$ then $\frac{\partial e^*}{\partial s_2} < 0$. This case corresponds to the situation in which firm 1 does not expel the competitive fringe. On the other hand, if $s_1^0 + g(e^M) \geq \tilde{s}_1(s_2)$, then call \bar{s}_2 the minimum value of s_2 such that $\tilde{s}_1(s_2) = s_1^0 + g(e^M)$. This is the minimum value of s_2 such that firm 1 optimally chooses an effort level such that its quality level ends up being $\tilde{s}_1(s_2)$. By (11) at \bar{s}_2 , $\frac{\partial \pi_1(s_2)}{\partial s_1} \frac{\partial s_1}{\partial e} > \alpha$. On the other hand, call $\bar{\bar{s}}_2$ the value of s_2 such that

$$\frac{\partial \pi_1(s_2)}{\partial s_1} \frac{\partial s_1}{\partial e} = \alpha$$

Now for $s_2 \in [\bar{s}_2, \bar{\bar{s}}_2]$, it is the case that $\frac{\partial e^*}{\partial s_2} > 0$. To see this, notice that for $s_2 \in [\bar{s}_2, \bar{\bar{s}}_2]$,

$$\frac{\partial \pi_1(s_2)}{\partial s_1} \frac{\partial s_1}{\partial e} > \alpha$$

and

$$\frac{\partial \pi_1^M}{\partial s_1} \frac{\partial s_1}{\partial e} < \alpha$$

Hence, the solution is $e^* = g^{-1}(\tilde{s}_1(s_2) - s_1^0)$, that is, firm 1 chooses e so that its quality level ends up being $\tilde{s}_1(s_2)$. Since, we know that $\frac{\partial \tilde{s}_1(s_2)}{\partial s_2} > 0$, it must be the case that $\frac{\partial e^*}{\partial s_2} > 0$ for $s_2 \in [\bar{s}_2, \bar{\bar{s}}_2]$.

The main result of the analysis is summarized in Proposition (??):

Proposition 1 *If $s_2 \in \left[\frac{c_2}{\theta}, \frac{2c_2}{\theta}\right]$, and there is some s_2 such that $s_1^0 + g(e^M) \geq \tilde{s}_1(s_2)$ there is an interval of values of s_2 such that firm 1 increases its investment in quality upgrading when s_2 increases.*

The existence of the interval of values of s_2 such that within that interval $\frac{\partial e^*}{\partial s_2} > 0$ allows for the possibility of an inverted-U relationship between competitive pressure and innovation. In fact, for $s_2 < \bar{s}_2$, producers of the low-quality variety do not operate, and thus $\frac{\partial e^*}{\partial s_2} = 0$, and for $s_2 > \bar{\bar{s}}_2$, $\frac{\partial e^*}{\partial s_2} < 0$. Hence, the existence of the interval $[\bar{s}_2, \bar{\bar{s}}_2]$ implies that the relationship between competitive pressure is positive, then negative.

3.1 Example

The following example illustrates the relationship between s_2 and e^* , given the value s_1^0 . To see this, assume that $m = 1$, $\underline{\theta} = 0$, $\bar{\theta} = 1$, and $g(e) = \sqrt{e}$. Regarding costs, assume that $c_1 = 0.3$ and that $c_2 = 0.25$. Then, for $s_2 \in \left[\frac{1}{4}, \frac{1}{2}\right]$ we could encounter an interval of values of s_2 such that e^* increases when s_2 does. In particular, this is the case for $s_2 \in [0.405, 0.415]$. Figures 1, 2, and 3 illustrate this, displaying firm 1's optimal choice for $s_2 = \{0.405, 0.415, 0.43\}$ respectively. In the figures, the blue line represents $\frac{\partial \pi_1(s_2)}{\partial s_1} \frac{\partial s_1}{\partial e}$, whereas the red line represents $\frac{\partial \pi_1^M}{\partial s_1} \frac{\partial s_1}{\partial e}$. Notice that the discontinuity moves to the right as the value of s_2 increases. When $s_2 = 0.405$ the optimum is at the discontinuity, and so is when $s_2 = 0.415$. Finally notice that for $s_2 = 0.43$ the optimal value of e is lower than when $s_2 = 0.415$, giving rise to the inverted-U relationship between competitive pressure and incentives to innovate.

4 Conclusions

In this paper, I propose a model of vertical product differentiation where a monopolist in the production of a higher-quality good faces competition against a number of firms that produce a lower-quality product. The model is used to analyze the high-quality producer's incentives to invest in raising the quality of its product. While the relationship is typically negative, I find that there are parameter values such that it is possible to obtain an inverted-U-shaped relationship between competitive pressure and incentives to innovate.

This model may be reinterpreted to analyze the incentives that formal firms in developing countries have to innovate, given that they compete against firms in the informal sector, which typically produce lower-quality versions of the products marketed by formal firms.

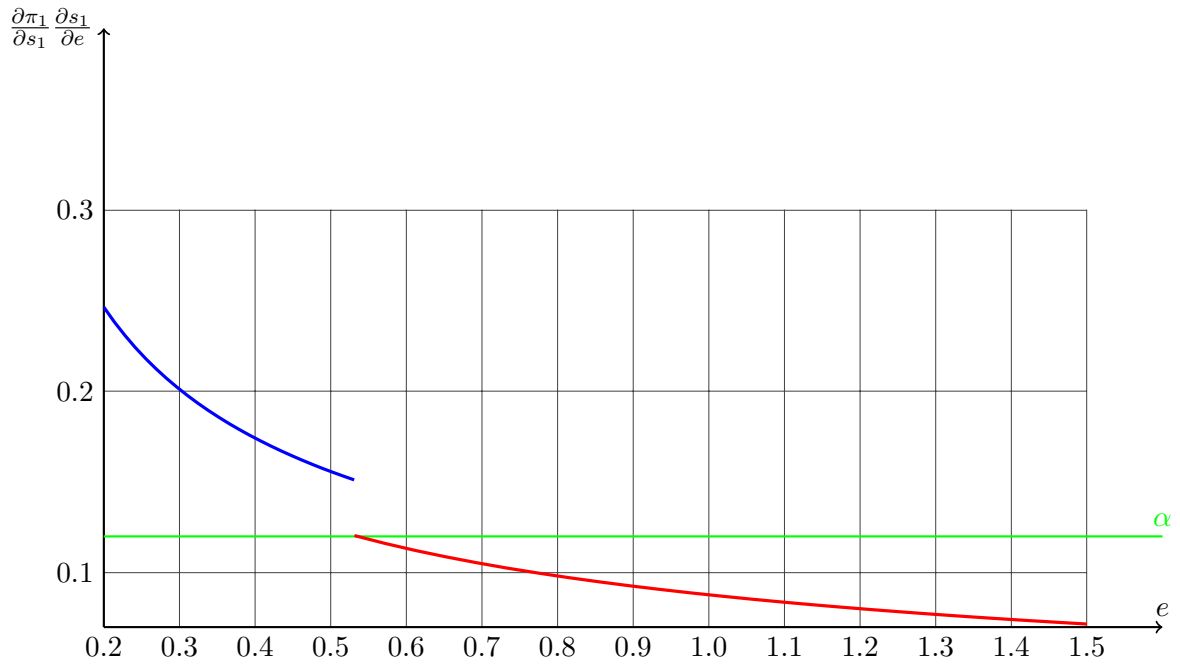


Figure 1: Incentives to innovate when $\alpha = 0.12$, $c_1 = 0.3$, $c_2 = 0.25$, $s_1 = 0.55$, and $s_2 = 0.405$

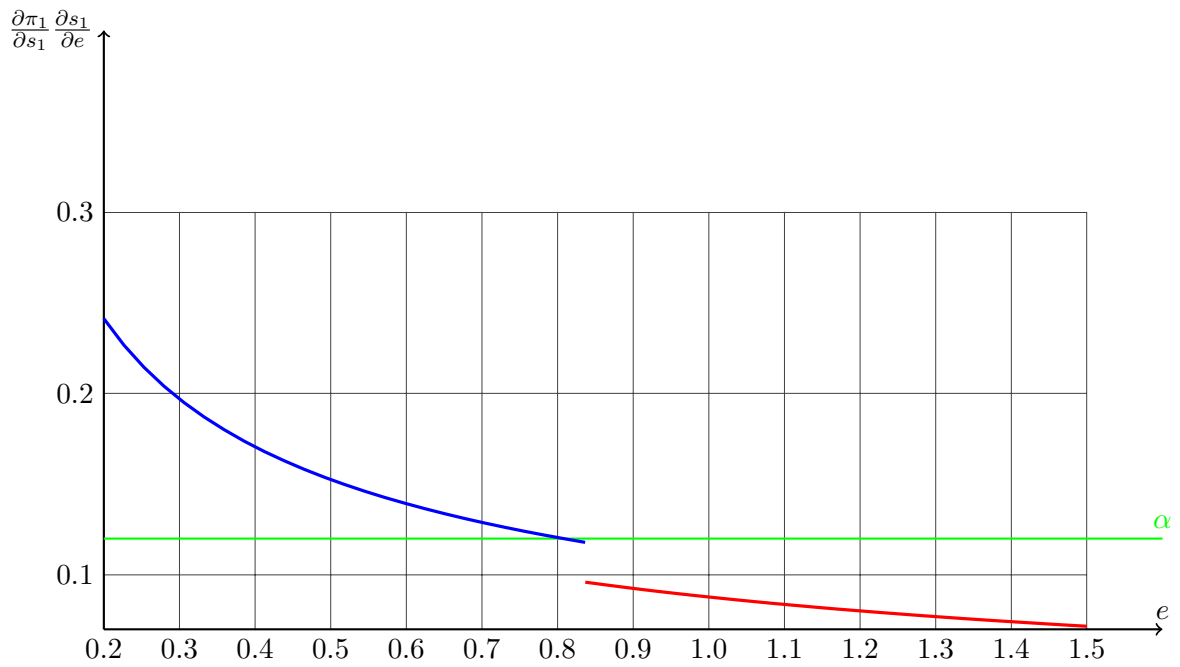


Figure 2: Incentives to innovate when $\alpha = 0.12$, $c_1 = 0.3$, $c_2 = 0.25$, $s_1 = 0.55$, and $s_2 = 0.415$

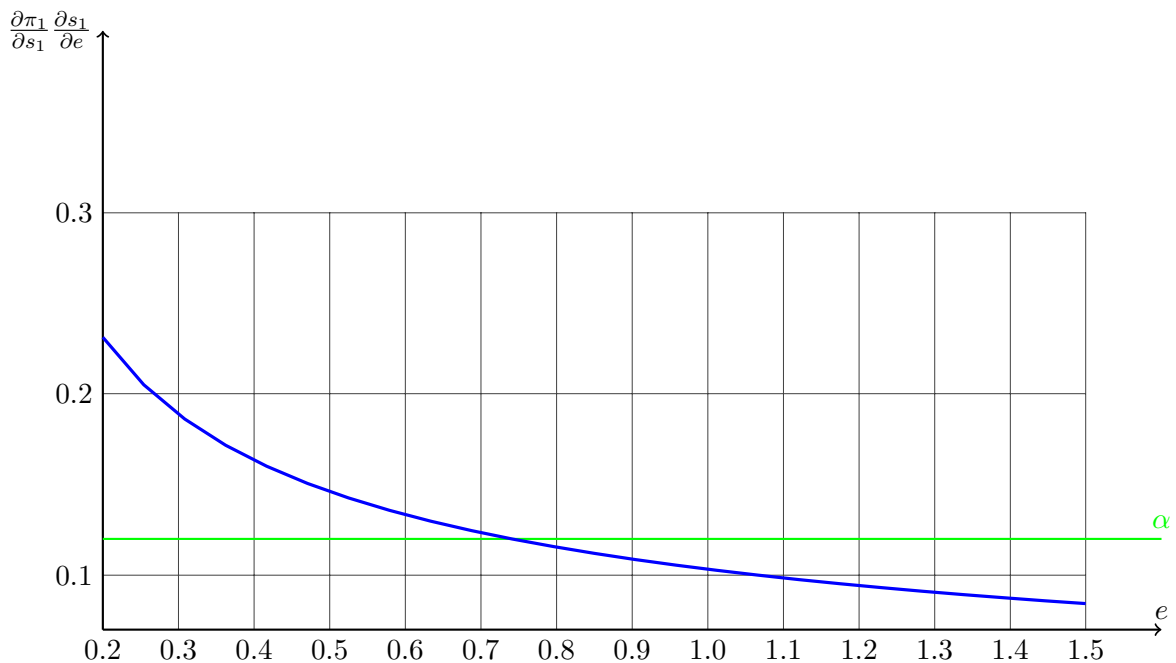


Figure 3: Incentives to innovate when $\alpha = 0.12$, $c_1 = 0.3$, $c_2 = 0.25$, $s_1 = 0.55$, and $s_2 = 0.43$

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